Announcement: Midterm the week of Feb 11 – 15
(To be determined)

Quantifiers (Ch 2.7, 2.8)

Examples: $P(n)$: "n^2-1 is a multiple of 8"

Q: "For every integer n, n^2-1 is a multiple of 8."

R: "There exists an integer n s.t. n^2-1 is a multiple of 8."

$P(n)$ is an open sentence. Its truth value depends on n.

Q is false. For example, if we take $n=2$ then n^2-1 is not a multiple of 8.

R is true. For example, $n=3 \rightsquigarrow n^2-1$ is a multiple of 8

$\forall$ = "for every" **Q**: "$\forall n \in \mathbb{Z},\ 8 \mid n^2-1$" (false)

$\exists$ = "exists" **R**: "$\exists n \in \mathbb{Z},\ 8 \mid n^2-1$" (true)

Comment: Usually if no clear quantifier is given, we mean $\forall$. Ex. "Let n be an integer. Then ..."

$\equiv$ "$\forall n \in \mathbb{Z}, \ldots$"

<u>Exercise:</u> Convert to quantifiers

(i) For every integer a and integer b, ab is even if and only if a is even or b is even

(ii) If $n \equiv 3 \bmod 4$ then there do not exist integers a and b s.t. $a^2+b^2=n$

(iii) For every odd integer n, $8 \mid n^2-1$

(iv) There exist positive integers a & b such that $a>1$ and $b>1$ and $ab=n$.

<u>Answers:</u> (i) P: $\forall a \in \mathbb{Z}, \forall b \in \mathbb{Z}, \ (2 \mid ab \iff (2 \mid a) \vee (2 \mid b))$

(ii) Q(n): $n \equiv 3 \bmod 4 \implies \sim \exists a, b \in \mathbb{Z} \ (a^2+b^2=n)$

$n \equiv 3 \bmod 4 \implies \nexists a, b \in \mathbb{Z} \ (a^2+b^2=n)$

(iii) R(n): $\forall n \in \{2x+1 \mid x \in \mathbb{Z}\}, (8 \mid n^2-1)$

$\equiv \forall n \in \mathbb{Z}, ((\exists x \in \mathbb{Z}, n=2x+1) \implies 8 \mid n^2-1)$

(n is odd $\implies 8 \mid n^2-1$)

(iv) S(n): $\exists a, b \in \mathbb{N}, (a>1 \wedge b>1 \wedge ab=n)$

Negating Quantifiers:

Imagine a sentence "$\forall x \in S,\ P(x)$"

The negation $\sim \forall x \in S, P(x)$

is logically equivalent to $\exists x \in S (\sim P(x))$.

Likewise, $\sim \exists x \in S, P(x) \equiv \forall x \in S, (\sim P(x))$

Another perspective: $\forall x \in \{5, 7, 9\},\ P(x)$

$P(5) \wedge P(7) \wedge P(9)$

negation s. by De Morgan's Law

$\exists x \in \{5, 7, 9\},\ \sim P(x)$

$\sim P(5) \vee \sim P(7) \vee \sim P(9)$